# Inferring Causal Dependencies between Chaotic Dynamical Systems from Sporadic Time Series

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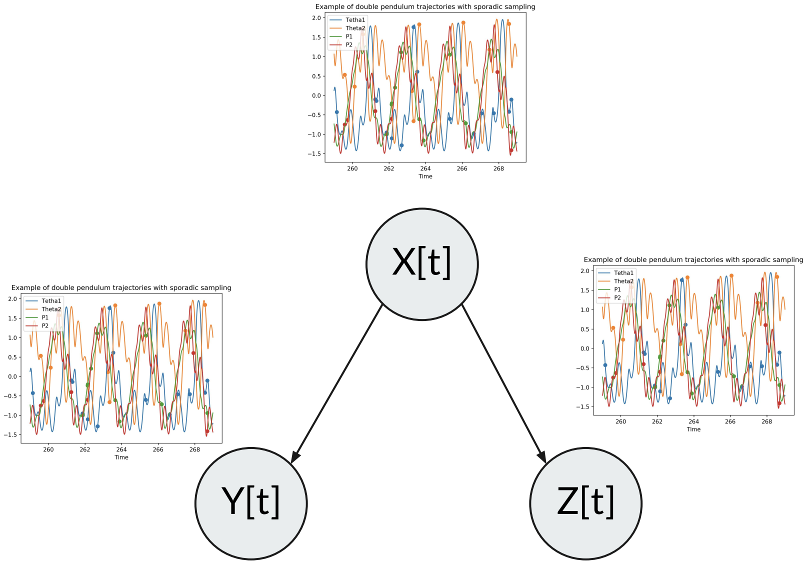
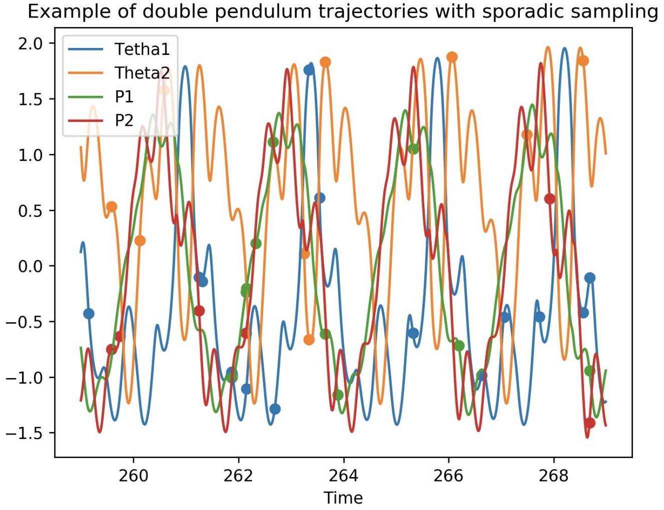
Link to the paper

# Inferring causal dependencies from time series

**Problem setup:** Inferring causal dependency between two multivariate time series X[t] and Y[t], typically not regularly spaced.

. Eventually, we want to uncover the full causal structure between several sporadically observed time series.

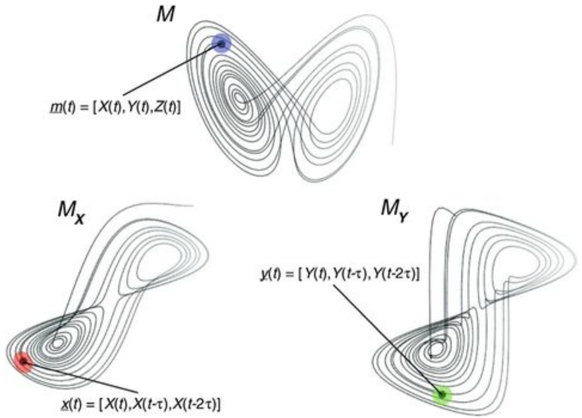


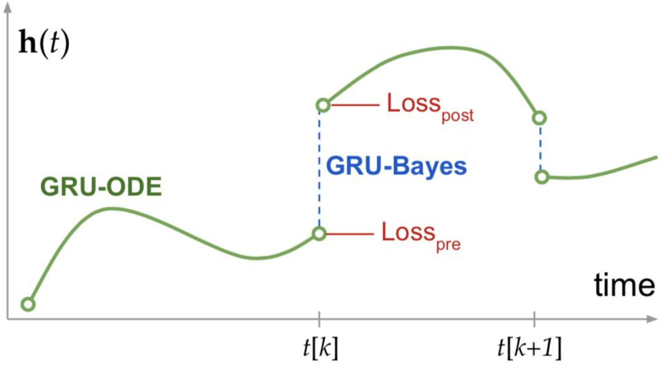
# Convergent cross mappings and GRU-ODE-Bayes

**Methods:** To infer causal direction between time series, we use a combination of convergent cross mappings (CCM) [1] and GRU-ODE-Bayes [2].

**CCM :** *X causes Y if we can recover X from Y*
A key step in the CCM methodology is to compute the delay embedding of both time series: X'[t] and Y'[t]. However, when the data is only sporadically observed at irregular intervals, the probability of observing the delayed samples $X_i[t], X_i[t-\tau], \ldots, X_i[t-k\tau]$ is vanishing for any t.




**GRU-ODE-Bayes:** We propose to impute the sporadic time series by learning its governing ODE dynamics and then use those interpolated samples to compute the delay embeddings of both processes. In particular, we use GRU-ODE-Bayes {2}, a filtering technique that extends Neural ODEs. The method jointly learns the ODE driving the data and computes the filtered probability of future samples conditioned on previous ones, in continuous time.

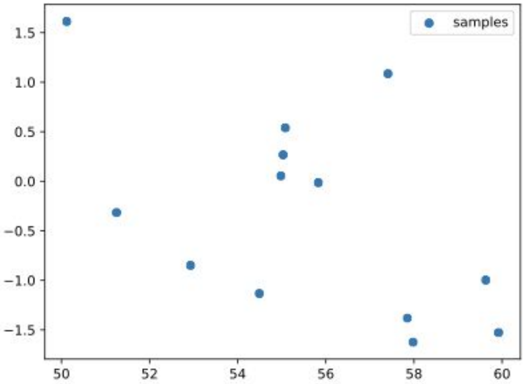




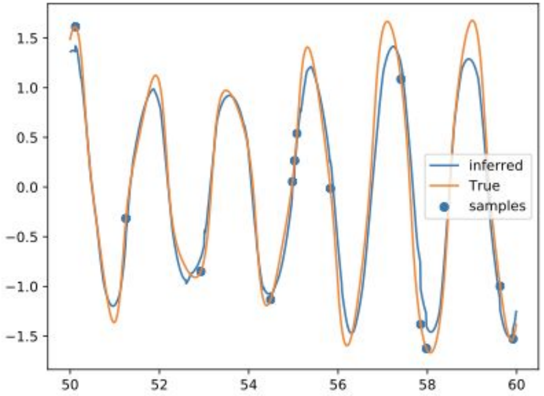
[1] Sugihara, G., May, R., Ye, H., Hsieh, C.-h., Deyle, E., Fogarty, M., and Munch, S. Detecting causality in complex ecosystems. science, 338(6106):496–500, 2012

[2] De Brouwer, E., Simm, J., Arany, A., and Moreau, Y. Gru-ode-bayes: Continuous modeling of sporadically observed time series. In Advances in Neural Information Processing Systems, pp. 7377–7388, 2019.

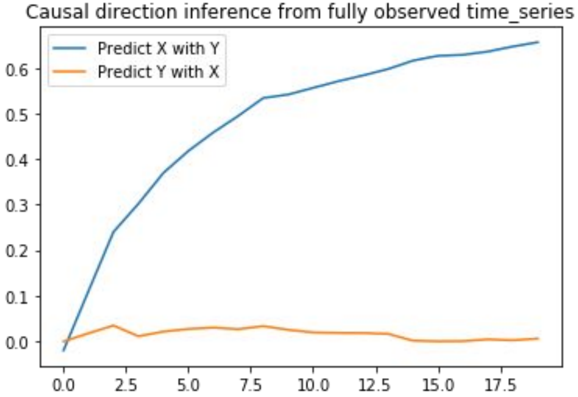
# A glimpse at the full pipeline




Sporadic time series data




GRU-ODE-Bayes reconstruction

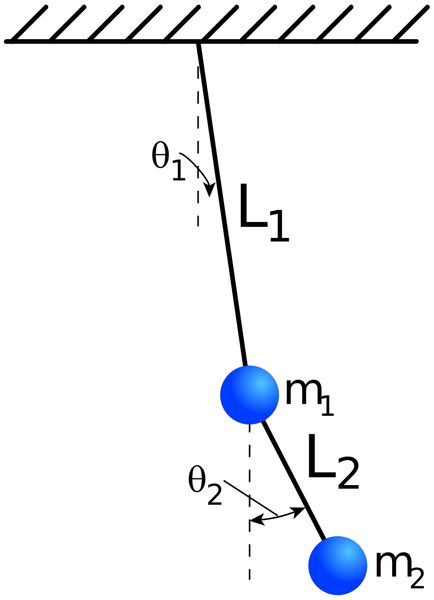



Causal direction inference (CCM)

**Pipeline:** From the sporadic time series, we learn a filtered continuous time reconstruction with GRU-ODE-Bayes. We use this reconstruction to compute a delay embedding and infer the causal directionality between time series using CCM.

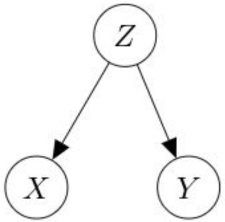
# Case study : double pendulums

**Experiments:** We evaluate the performance of our approach on samples from the trajectories of three double pendulums. The metric is the correlation between reconstructed delay embedding manifolds. When positive, it indicates a causal link between time series. We consider 2 main cases. In the second, we tackle to challenging problem of confounding. Our approach is the only one among the compared baselines to recover the correct generating process.




**Case 1**


**Case 2**




| CASE | DIRECTION | LINEAR | GP | MVGP | OURS |
|---|---|---|---|---|---|
| CASE 1 | $X \leftarrow Y$ | $0.001 \pm 0.006$ ✓ | $-0.003 \pm 0.005$ ✓ | $-0.014 \pm 0.05$ ✓ | $0.0017 \pm 0.005$ ✓ |
| | $\mathbf{X} \rightarrow \mathbf{Y}$ | $0.000 \pm 0.004$ ✗ | $0.003 \pm 0.005$ ✗ | $-0.002 \pm 0.037$ ✗ | $\mathbf{0.209}^{*} \pm 0.037$ ✓ |
| CASE 2 | $X \leftarrow Y$ | $-0.0005 \pm 0.005$ ✓ | $-0.001 \pm 0.008$ ✓ | $-0.009 \pm 0.25$ ✓ | $0.0001 \pm 0.007$ ✓ |
| | $X \rightarrow Y$ | $0.001 \pm 0.005$ ✓ | $0.001 \pm 0.003$ ✓ | $-0.007 \pm 0.019$ ✓ | $-0.019 \pm 0.06$ ✓ |
| | $X \rightarrow Z$ | $0.003 \pm 0.007$ ✓ | $-0.001 \pm 0.002$ ✓ | $0.001 \pm 0.087$ ✓ | $-0.003 \pm 0.003$ ✓ |
| | $\mathbf{Z} \rightarrow \mathbf{X}$ | $0.001 \pm 0.007$ ✗ | $\mathbf{0.082} \pm 0.002$ ✓ | $-0.013 \pm 0.033$ ✗ | $\mathbf{0.698} \pm 0.299$ ✓ |
| | $Y \rightarrow Z$ | $0.002 \pm 0.006$ ✓ | $0.001 \pm 0.003$ ✓ | $0.003 \pm 0.015$ ✓ | $0.003 \pm 0.012$ ✓ |
| | $\mathbf{Z} \rightarrow \mathbf{Y}$ | $0.002 \pm 0.005$ ✗ | $0.003 \pm 0.003$ ✗ | $0.0034 \pm 0.091$ ✗ | $\mathbf{0.096} \pm 0.048$ ✓ |